

SHOULD SPARTICLE MASSES UNIFY AT THE GUT SCALE?¹

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Abstract

Gauge and Yukawa (for the third family) coupling unification seem to be the best predictions of the grand-unified theories (GUTs). In supersymmetric GUTs, one also expects that the sparticle masses unify at the GUT scale (for sparticles embedded in the same GUT multiplet). I show under what circumstances GUTs do not lead to sparticle mass unification. In particular, I give examples of SU(5) and SO(10) SUSY GUTs in which squarks and sleptons of a family have different tree-level masses at the unification scale. The models have interesting relations between Yukawa couplings. For example, I present an SO(10) GUT that allows for a large ratio of the top to bottom Yukawas, accounting for the large m_t/m_b . The splittings can also be induced in the Higgs soft masses and accommodate the electroweak breaking.

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1 Introduction

Several reasons persuade us to study supersymmetric (SUSY) grand-unified theories (GUTs) and their low-energy predictions. Two motivations that I find very appealing are:

- Gauge coupling unification: Taking the values of the three gauge couplings at the weak scale and extrapolating (using the RGEs) to high energies, one finds that they meet at a scale $M_G \sim 10^{16}$ GeV [1, 2].
- The matter field multiplets, which under the standard model group look as in arbitrary representations, can be embedded in few multiplets of the GUT [3]. For example in SU(5), the matter fields (of a given generation) are embedded in the $\mathbf{\bar{5}}$ and $\mathbf{10}$, and in SO(10) they surprisingly fit in a single $\mathbf{16}$.

The above hints strengthen the hypothesis that (i) there is a manifest GUT at $M_G \sim 10^{16}$ GeV and (ii) the effective theory below M_G is just the supersymmetric standard model (MSSM) [1] (desert hypothesis). The desert hypothesis acts as a microscope that magnifies by 14 orders of magnitude and allows us to look at Planckian distances; it allows us to translate theoretical ideas near the Planck mass into low-energy predictions. There are many more parameters in addition to gauge couplings to be used as probes of SUSY GUTs: fermion and sparticle masses and mixing angles add up to a grand total of 110 physical parameters just in the supersymmetric flavour sector. According to the desert hypothesis, each of these parameters carries direct information about the structure of the theory at Planckian distances.

Here I will focus on the sparticle masses [4]. It is often assumed that the sparticles are degenerate at M_G (universality); it was motivated from the need to suppress flavour-violating processes [1]. Nevertheless, sparticles in different multiplets of the unified group have no symmetry reason to be degenerate at M_G and even if they are assumed to be degenerate at the Planck scale (as in minimal supergravity theories) their different interactions will split them by the time they reach the unification scale [5, 6, 7]. These splittings are typically large due to the large size of the representations in unified groups [6, 7].

Although this strong form of universality is not very realistic in GUTs, it is widely believed that a more restricted form is always valid: sparticles belonging to the same multiplet are degenerate at the unification scale. This is considered a direct consequence of unification, which will be experimentally checked if sparticles are discovered at the LHC and NLC and their masses are known with some precision.

I will show that, because the GUT group is spontaneously broken, there is no good reason for this belief [4]. Sparticles, such as \tilde{b}_R and $\tilde{\tau}_L$, which by virtue of their gauge and family quantum numbers can be grouped into an irreducible SU(5) multiplet, do not necessarily originate from *one* such multiplet; they could come about from a linear combination of several multiplets. This can produce non-degeneracy among sparticles of the same generation and complementary SU(5) quantum numbers². I will show that these sparticle splittings occur precisely in theories (and for the same reasons) that produce interesting and desirable relations among fermion masses.

²These are sparticles whose combined quantum numbers complete an irreducible SU(5) multiplet.

2 Non-unified sparticle masses in an SU(5) model

The model [4] is a minimal extension of the SU(5) SUSY GUT [1]. Consider an SU(5) theory with just the third generation consisting of a $\bar{\mathbf{5}}_1$ and $\mathbf{10}_1$, the usual Higgs fiveplet \mathbf{H} and antifiveplet $\bar{\mathbf{H}}$, and the adjoint $\mathbf{24}$ that breaks SU(5) down to SU(3)×SU(2)×U(1) at the unification scale M_G by acquiring a vacuum expectation value (VEV) that points in the hypercharge direction:

$$\langle \mathbf{24} \rangle = V_{24} \mathbf{Y} \equiv V_{24} \text{diag}(2, 2, 2, -3, -3). \quad (1)$$

The bottom and tau masses are given by the superpotential

$$W = h \mathbf{10}_1 \bar{\mathbf{H}} \bar{\mathbf{5}}_1, \quad (2)$$

and are equal at the unification scale [8]. Now add an extra fiveplet and antifiveplet denoted by $\mathbf{5}$ and $\bar{\mathbf{5}}_2$ with the following couplings:

$$W = \mathbf{5} [M \bar{\mathbf{5}}_1 + \lambda \mathbf{24} \bar{\mathbf{5}}_2] + h \mathbf{10}_1 \bar{\mathbf{H}} \bar{\mathbf{5}}_1, \quad (3)$$

where M is near the unification mass. One linear combination of $\bar{\mathbf{5}}_1$ and $\bar{\mathbf{5}}_2$ will acquire a large mass of order $\sim M_G$. The orthogonal combination will be part of the low-energy spectrum. It contains the right-handed bottom quark and the tau lepton doublet which are denoted by D^c and L respectively; because the hypercharges of D^c and L differ, it follows from eqs. (1) and (3) that they will be *different* linear combinations of the corresponding states in $\bar{\mathbf{5}}_1$ and $\bar{\mathbf{5}}_2$:

$$\begin{pmatrix} D^c \\ L \end{pmatrix} = -\sin \theta_Y \bar{\mathbf{5}}_1 + \cos \theta_Y \bar{\mathbf{5}}_2, \quad (4)$$

where

$$\sin \theta_Y = \frac{\rho \mathbf{Y}}{\sqrt{1 + \rho^2 \mathbf{Y}^2}}, \quad (5)$$

with $\rho = \lambda V_{24}/M$.

Since $\bar{\mathbf{5}}_1$ and $\bar{\mathbf{5}}_2$ are in different representations of SU(5), they have, in general, different soft SUSY-breaking masses at M_G [7]:

$$\mathcal{L}_{soft} = m_1^2 |\bar{\mathbf{5}}_1|^2 + m_2^2 |\bar{\mathbf{5}}_2|^2. \quad (6)$$

Since the light combination is given by (4), one has

$$\begin{aligned} m_{b_R}^2 &= m_2^2 + s_{b_R}^2 (m_1^2 - m_2^2), \\ m_{\tau_L}^2 &= m_2^2 + s_{\tau_L}^2 (m_1^2 - m_2^2), \end{aligned} \quad (7)$$

where $b_R \in D^c$, $\tau_L \in L$ and s_a is given by

$$s_a = \sin \theta_{Y_a} = \frac{\rho Y_a}{\sqrt{1 + \rho^2 Y_a^2}}, \quad (8)$$

and Y_a is the hypercharge of a . Therefore, the squark and slepton masses differ at M_G ; their fractional mass-splitting, for $\Delta > 0$, is given by

$$\frac{m_{\tau_L}^2 - m_{b_R}^2}{m_{\tau_L}^2} = \frac{s_{\tau_L}^2 - s_{b_R}^2}{\Delta + s_{\tau_L}^2}, \quad (9)$$

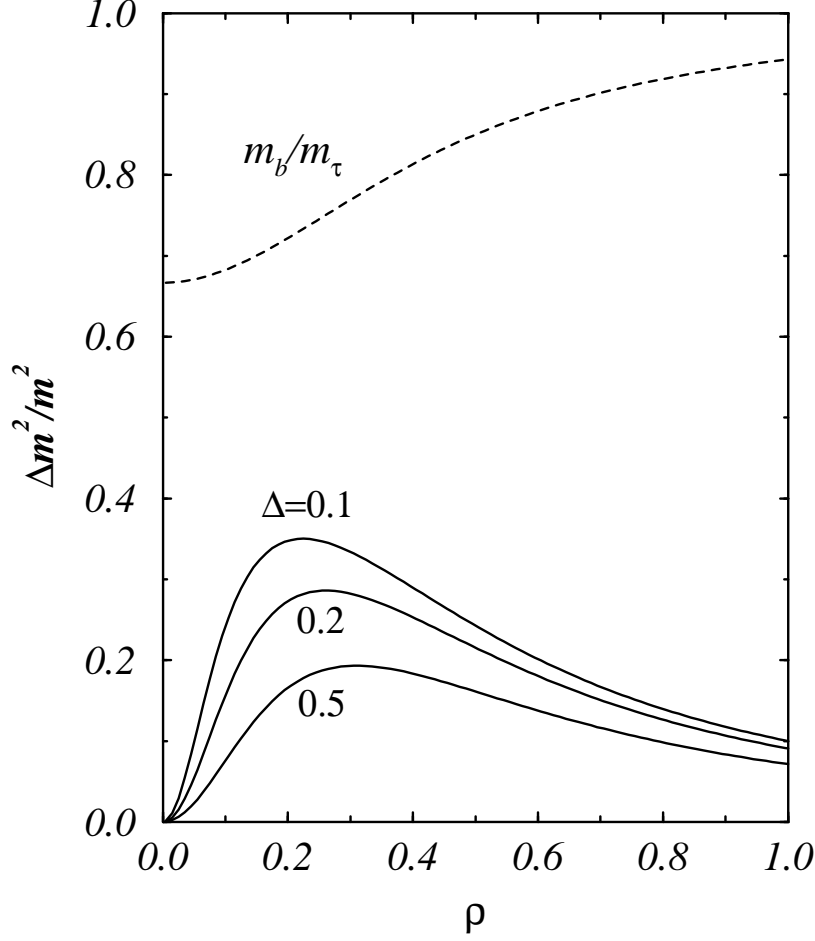


Figure 1: Scalar mass-splitting $\Delta m^2/m^2 \equiv (m_{\tilde{\tau}_L}^2 - m_{\tilde{b}_R}^2)/m_{\tilde{\tau}_L}^2$ as a function of $\rho = \lambda V_{24}/M$ and for different values of $\Delta = m_2^2/(m_1^2 - m_2^2)$. The dashed line corresponds to the ratio m_b/m_τ .

where $\Delta = m_2^2/(m_1^2 - m_2^2)$. Eq. (9) is plotted in Fig. 1. One can see that a mass-splitting of $\sim 30\%$ can be obtained.

The fermion masses arise from the Yukawa coupling $\mathbf{10}_1 \bar{\mathbf{H}} \bar{\mathbf{5}}_1$:

$$\begin{aligned} m_b &= h s_{b_R} \langle \bar{\mathbf{H}} \rangle, \\ m_\tau &= h s_{\tau_L} \langle \bar{\mathbf{H}} \rangle, \end{aligned} \quad (10)$$

which leads to the ratio between the bottom and tau mass

$$\frac{m_b}{m_\tau} = \frac{s_{b_R}}{s_{\tau_L}} = \frac{2}{3} \sqrt{\frac{1+9\rho^2}{1+4\rho^2}}. \quad (11)$$

This ratio tends to $2/3$ and 1 in the small and large ρ limit respectively. From eqs. (9) and (11), one can see that the scalar mass-splitting is correlated to the fermion one. This is shown in Fig. 1, where the dashed line represents the ratio m_b/m_τ . The maximum values for the scalar mass-splitting correspond to $m_b/m_\tau \sim 0.7-0.8$. Thus this model, although it is a minimal perturbation of the SU(5) SUSY GUT [1], easily accommodates values of m_b/m_τ

that are between $2/3$ and 1 . As a consequence, the strong constraints on the top mass that arise from bottom–tau unification [9] can be relaxed.

It is now easy to see how the sparticle and particle splittings came about in this model. Although the right-handed bottom and the tau lepton doublet – by virtue of their family and gauge quantum numbers – appear to belong to the same $\bar{\mathbf{5}}$ of $SU(5)$, they in fact, because of their different hypercharges, came from two *different* linear combinations of a pair of $\bar{\mathbf{5}}$ s. This causes $SU(5)$ -breaking effects in sparticles and particles to be felt at the tree-level, since they occur at the very basic stage of defining the light states of the theory³.

The same idea can be implemented for the states in the decuplets, adding to the previous model an extra $\overline{\mathbf{10}}$ and $\mathbf{10}_2$. As shown in Fig. 2 of ref. [4], these splittings can be much larger than for the fiveplet since the differences in hypercharges are larger in the decuplet.

3 An $SO(10)$ model with large h_t/h_b

The previous example can be easily adapted to $SO(10)$; the Higgs are in the $\mathbf{45} \ni \mathbf{24}$ and $\mathbf{10}_H \ni \{\mathbf{H}, \bar{\mathbf{H}}\}$, and the matter fields are embedded in the $\mathbf{16}$ spinor representations. Here, however, I will consider a different scenario. Instead of adding an extra $\overline{\mathbf{16}}$ and $\mathbf{16}$, I will add a $\mathbf{10}$, $\mathbf{10}'$ and a $\mathbf{16}_H$ that gets a VEV of $\mathcal{O}(M_G)$:

$$W = \mathbf{10}' [M\mathbf{10} + \lambda \mathbf{16}_H \mathbf{16}] + h \mathbf{16} \mathbf{10}_H \mathbf{16}. \quad (12)$$

As in the previous model, the light quarks and leptons arise from the linear combination, $-\sin\theta\mathbf{10} + \cos\theta\mathbf{16}$, where now the mixing angle is (because only the $SU(5)$ singlet of $\mathbf{16}_H$ gets a VEV)

$$\sin\theta = \begin{cases} \rho/\sqrt{1+\rho^2} & \text{for } D^c \text{ and } L, \\ 0 & \text{for the rest of the fields,} \end{cases} \quad (13)$$

where $\rho = \lambda\langle\mathbf{16}_H\rangle/M$. Notice that only D^c and L are a mixture of both multiplets, the $\mathbf{10}$ and the $\mathbf{16}$, while the other particles come only from the $\mathbf{16}$. This is because under $SU(5)$ $\mathbf{10} = \mathbf{5} + \bar{\mathbf{5}}$ and it only contains states of gauge quantum numbers of the D^c and L . The scalar masses are split according to

$$m_a^2 = m_{\mathbf{16}}^2 + \sin^2\theta(m_{\mathbf{10}}^2 - m_{\mathbf{16}}^2), \quad (14)$$

which preserve $SU(5)$ invariance because the $\langle\mathbf{16}_H\rangle$ does not break this subgroup of $SO(10)$. The fermion masses in this model are proportional to $c_a \equiv [\cos\theta]_a$, the $\cos\theta$ of the field a . For the third family, one has

$$\frac{h_t}{h_b} = \frac{c_{t_R}c_{t_L}}{c_{b_R}c_{b_L}} = \frac{1}{c_{b_R}}, \quad (15)$$

which for large values of ρ ($M \ll \langle\mathbf{16}_H\rangle$) leads to

$$\frac{h_t}{h_b} \sim \rho \gg 1; \quad (16)$$

this accounts for the large mass difference $m_t \gg m_b$ without requiring a large ratio of the VEVs of the Higgs doublets.

The same mechanism of mass-splitting can be applied for the Higgs. In the minimal $SO(10)$ model the two light-Higgs doublets, H_1 and H_2 , are embedded in the $\mathbf{10}_H$; their soft

³ See ref. [10] for other uses of this mechanism.

masses are equal at M_G and a severe fine-tuning is required to get the correct electroweak symmetry breaking [11]. Let us introduce to the minimal model an extra $\mathbf{16}_H$ and two $\overline{\mathbf{16}}_S$; one of them gets a VEV of $\mathcal{O}(M_G)$:

$$W = \overline{\mathbf{16}} \left[M \mathbf{16}_H + \lambda \overline{\mathbf{16}}_H \mathbf{10}_H \right] + h \mathbf{16} \mathbf{10}_H \mathbf{16}. \quad (17)$$

In this model, one obtains

$$\begin{aligned} (m_{H_1}^2 - m_{H_2}^2)/m_{H_2}^2 &= \sin^2 \theta (m_{16}^2 - m_{10}^2)/m_{10}^2, \\ h_t/h_b &= 1/\cos \theta, \end{aligned} \quad (18)$$

where $\sin \theta = \rho/\sqrt{1+\rho^2}$ and $\rho = \lambda \langle \overline{\mathbf{16}}_H \rangle / M$. Since the M_P - M_G evolution [7] leads to $m_{16}^2 > m_{10}^2$, one finds the following nice correlation at M_G :

$$h_t > h_b \iff m_{H_1}^2 > m_{H_2}^2, \quad (19)$$

i.e., $m_{H_1}^2 > m_{H_2}^2$, which favours the electroweak breaking, is related with the fact that $m_t > m_b$.

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